ASV

M. Math 1st yr Mid-term 11-09-2013 Answer all the questions. 8x5 = 40 Time 3hrs

1) Give an example of a function $f : R^3 \to R$ that is not continuous, but has first order partial derivatives. Give full details.

2) Let $g : (-1,-1) \to R$ be an infinitely differentiable function. Let $F : (-1,-1) \times (-1,-1) \to R$ be defined by $F(x,y) = g(xy)$ for $-1 < x, y < 1$. Use induction to derive the formula, for non-negative integers l, m :

$$\frac{\partial^{l+m}}{\partial^l x \partial^m y}(0,0) = l! g^{(l)}(0) \;\; if \;\; l = m \;\; and \; 0 \; otherwise.$$

3) Find and classify the extreme values (if any) of $f(x,y) = x^2+y^2+x+y+xy$. State all the results required for your conclusions.

4) Let $H : [0,1]^4 \to R$ be defined by

$$H(x_1, x_2, x_3, x_4) = cos(\frac{\pi}{2}(x_1 + x_2)) + (x_3 + x_4 - 1)^3.$$

For $0 \leq x_i \leq 1$, $1 \leq i \leq 4$. Use **only** the Monotonicity theorem and other integration techniques described in the class, to prove that H is integrable. State the theorems you need in the proof.

5) Let $G : [0,1] \times [0,1] \times [0,1] \to [0,1]$ be defined by $G(x,y,z) = 1$, if $x = 0$ and y *is rational* and $G(x,y,z) = \frac{1}{q}$, if $x = \frac{p}{q}$, for relatively prime numbers p, q and y is rational. $G = 0$ at all other values of x, y. Show that G is integrable and $\int \int \int_{[0,1]^3} G \; dxdydz = 0$. State all the results required for your conclusion.